CSCI 1470

Eric Ewing

Monday, 1/27/25

Deep Learning

Day 3: Linear Regression, Perceptrons, and MNIST

Brown AI Safety Team

It's BAIST.

Our mission is to mitigate large-scale risks from advanced AI through technical research and effective policy interventions.

Apply to our policy and technical programs or become a member at BAIST.ai

DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning

DeepSeek-AI

research@deepseek.com

- Much less Supervised Fine Tuning (SFT) than previous models (e.g., GPT4)
- Uses Reinforcement Learning heavily (final part of this course)

Key Ideas Review

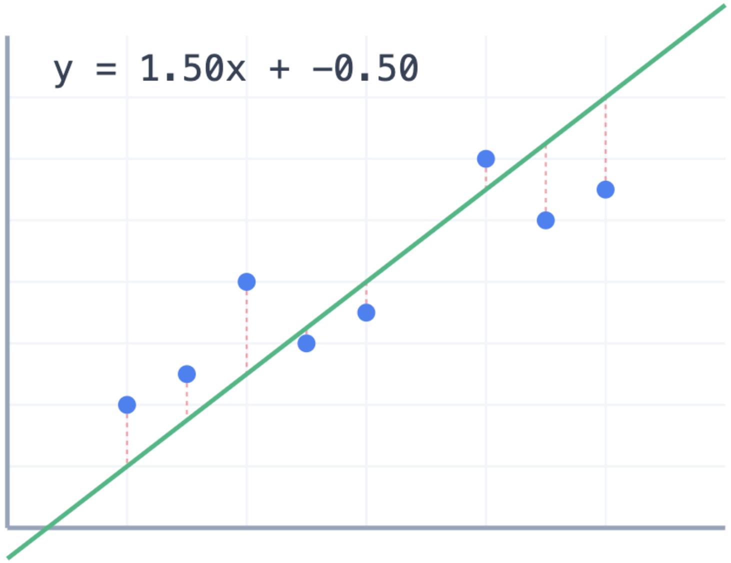
- How to represent Input/Output
  - Input/Output need to be numbers
  - Classification: Predicting categorical outputs (y is discrete)
  - Regression: Predicting numerical outputs (y is continuous)
- Supervised Learning
  - Goal of supervised learning: Find good approximation of data
  - Train on Training set
  - Evaluate on validation Set
  - But what we really care about is performance on test set

Today’s Goal: Learn about Perceptrons, the first building block of Neural Networks

- Optimization
- Perceptrons
- Introduction to MNIST

Linear Regression

$$y = mx + b$$



With 1 input feature, 2 ***parameters***

- m (slope)
- b (bias)

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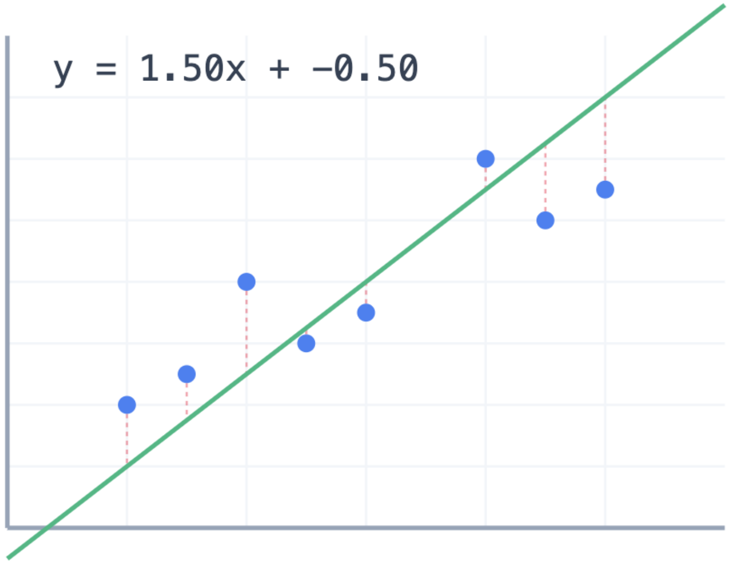

	Input Features			Output Target
	x_1	x_2	x_3	y
	Temperature	Sunny?	Day of Week	Profit
$x^{(1)}$	90	Yes	Sat	$200
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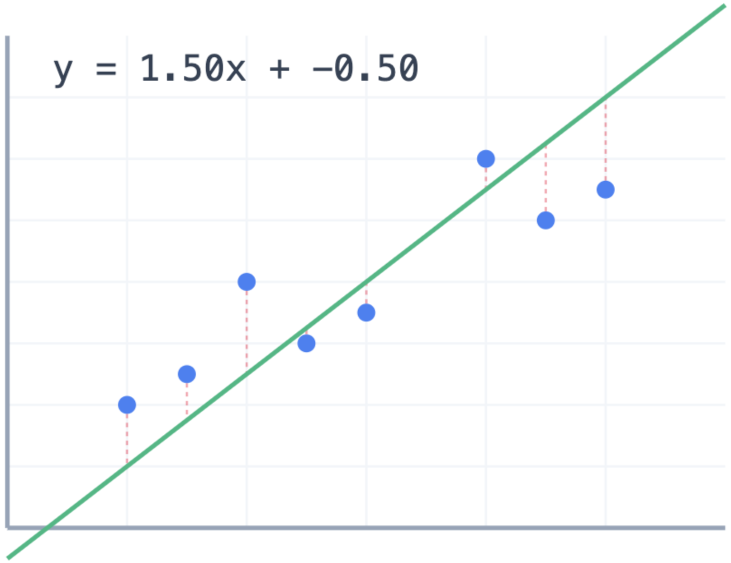

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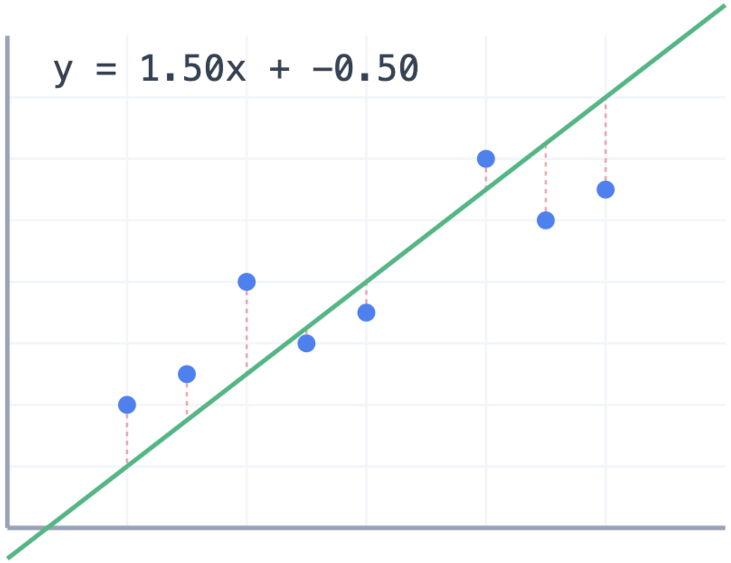

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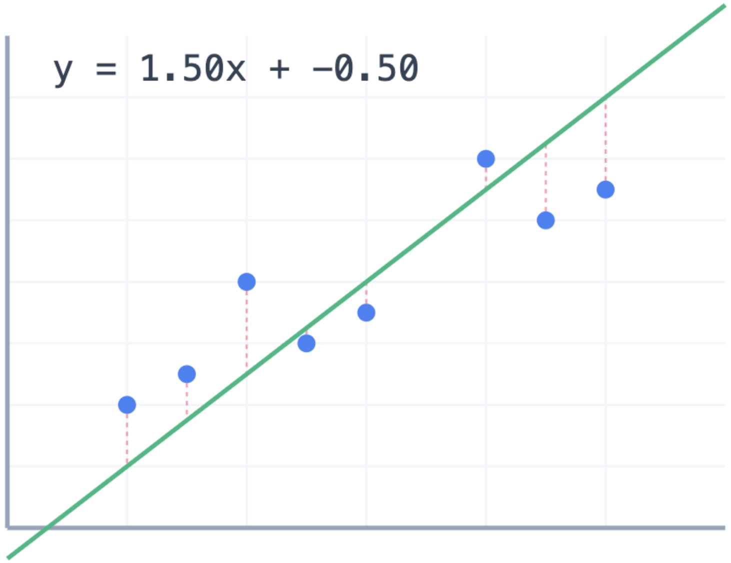
With multiple input features:

- Need a weight parameter w_i for each feature x_i
- $y = x_1^{(i)} \cdot w_1 + x_2^{(i)} \cdot w_2^{(i)} + \cdots + x_d^{(i)} \cdot w_d$
- Can be rewritten: $y = \vec{x} \cdot \vec{w}$

Linear Regression

How do we find optimal parameter values?

$$y = mx + b$$



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Option 1: Closed Form Solution

Goal: Minimize *Loss* function

Process:

- Find derivative (or gradient) of loss function
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weight vector $w \in \mathbb{R}^d$

Gradients

The gradient of a function *f* is a vector of partial derivatives:

$$\nabla f_\theta = [\frac{\partial f}{\theta_1}, \frac{\partial f}{\theta_2}, \frac{\partial f}{\theta_3}, \dots, \frac{\partial f}{\theta_d}]$$

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∇f_w tells us what happens to *f* with small adjustments to each parameter *w*

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$$\mathcal{L} = MSE = \frac{\sum_i^n (y_i - \vec{w}^T \vec{x})\text{^}2}{n}$$

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Matrix notation will make our lives easy!

$\mathbb{X} \in \mathbb{R}^{n \times d}, \mathbb{y} \in \mathbb{R}^{n}, \vec{\mathrm{w}} \in \mathbb{R}^{d}$

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$$(\mathbb{X}^T\mathbb{X})^{-1}(\mathbb{X}^T\mathbb{y}) = \vec{w}$$

Closed Form Solution

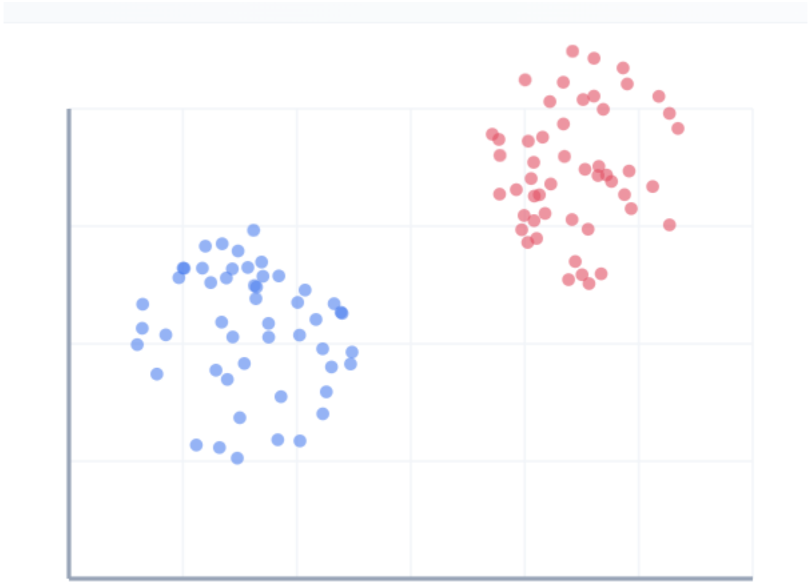
Advantages:

- Simple/fast to implement

Disadvantages:

- Need to invert: $(\mathbb{X}\mathbb{X}^T)^{-1}$
- Matrix inversion is $O(n^3)$
- $(\mathbb{X}\mathbb{X}^T)$ May not be invertible
- Doesn't necessarily exist for other models

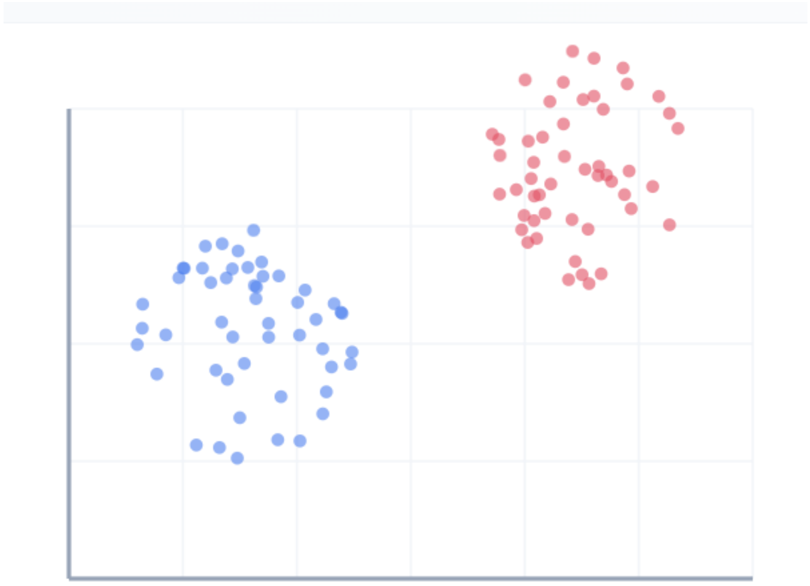
A Linear Classification Model



A Linear Classification Model

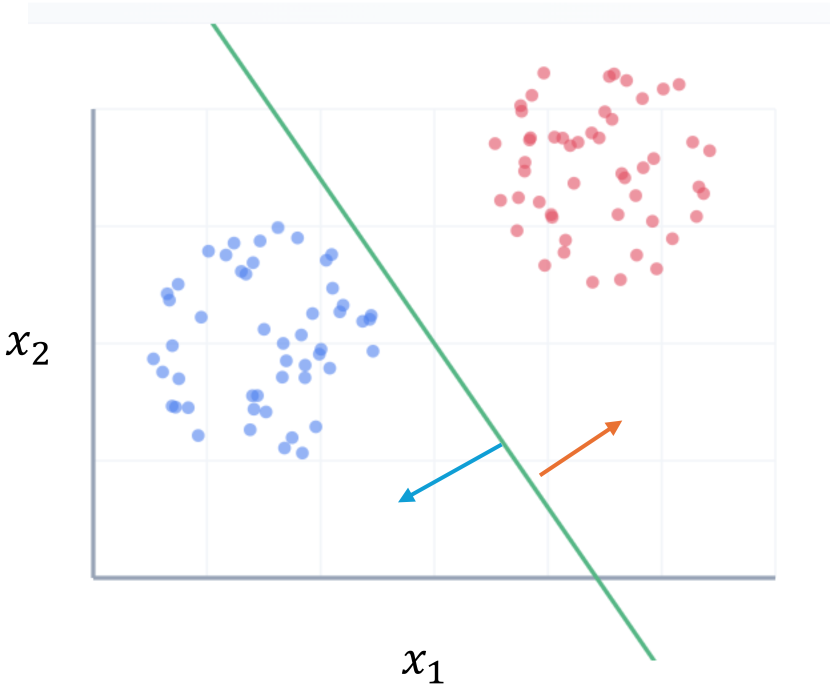
Linear Regression is a linear model for *regression*.

What's a natural way to make a linear *classifier*?



A Classifier

Everything above the line (or hyperplane in >2D) is classified as 1, everything below the line as 0



$$\hat{y} = w_1 \cdot x_1 + w_2 \cdot x_2 + b$$

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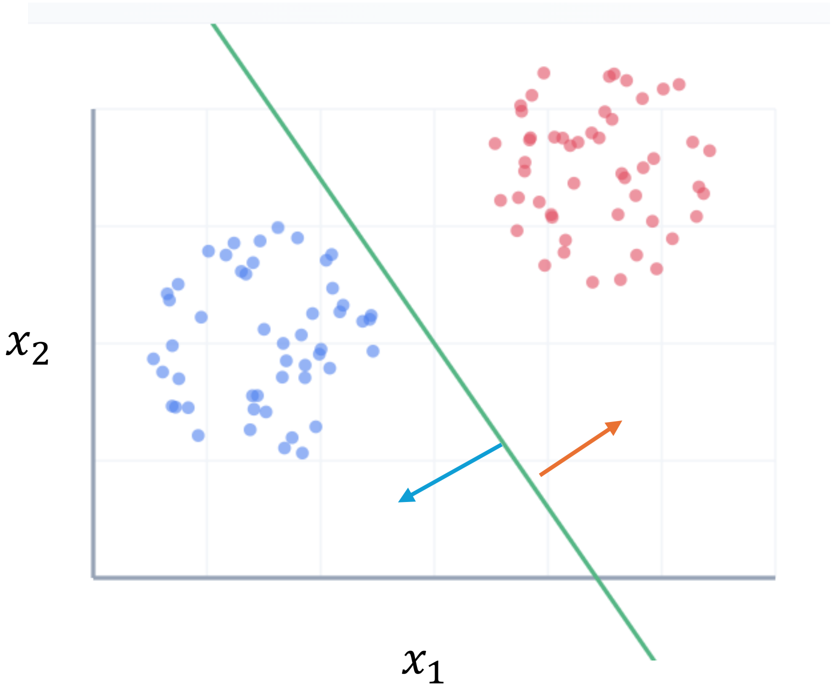
How can you tell if a point is above or below the line?



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If $\hat{y} = 0$, the point is **on** the line,
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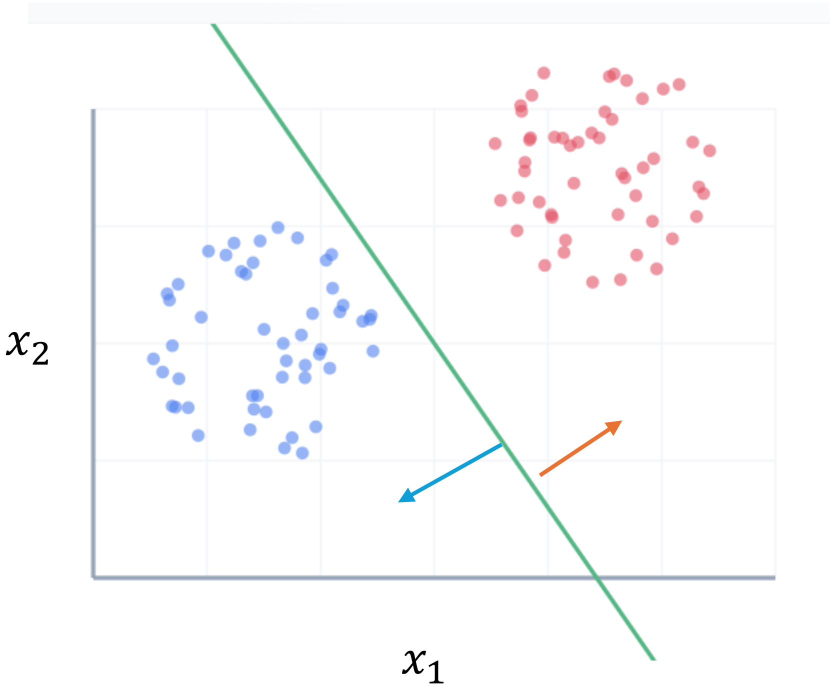

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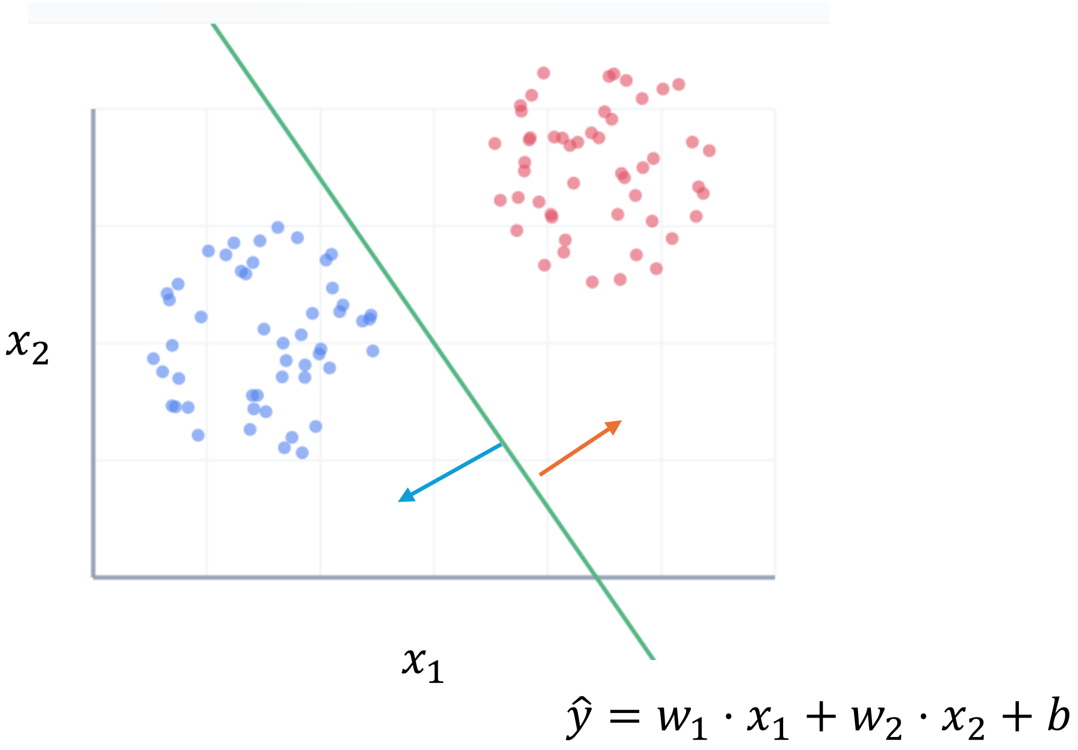
If $\hat{y} > 0$, predict 1.
If $\hat{y} \leq 0$, predict 0.

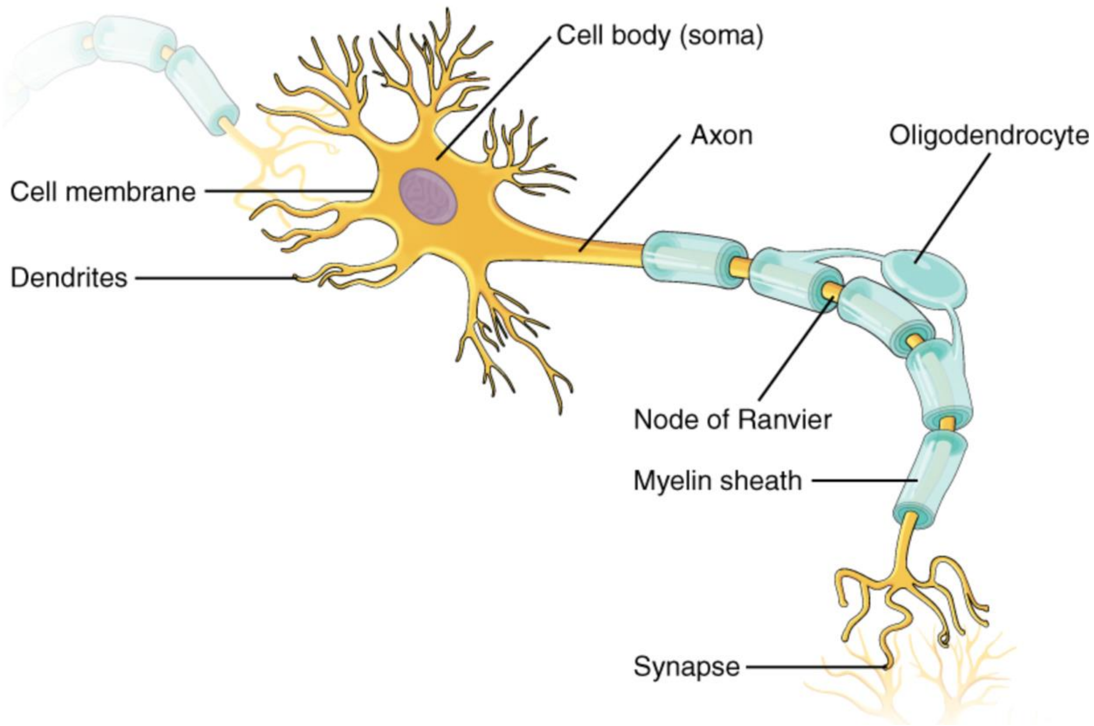


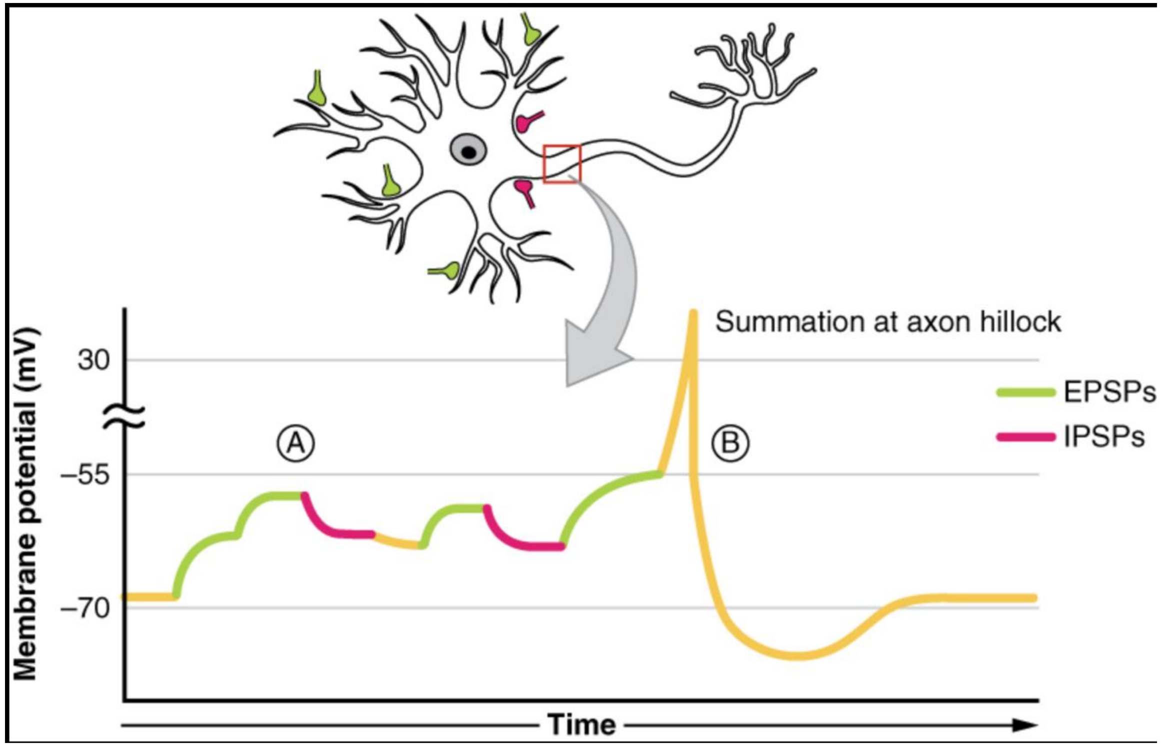
Perceptrons: A Linear Classifier

(Our first building block of Deep Learning)

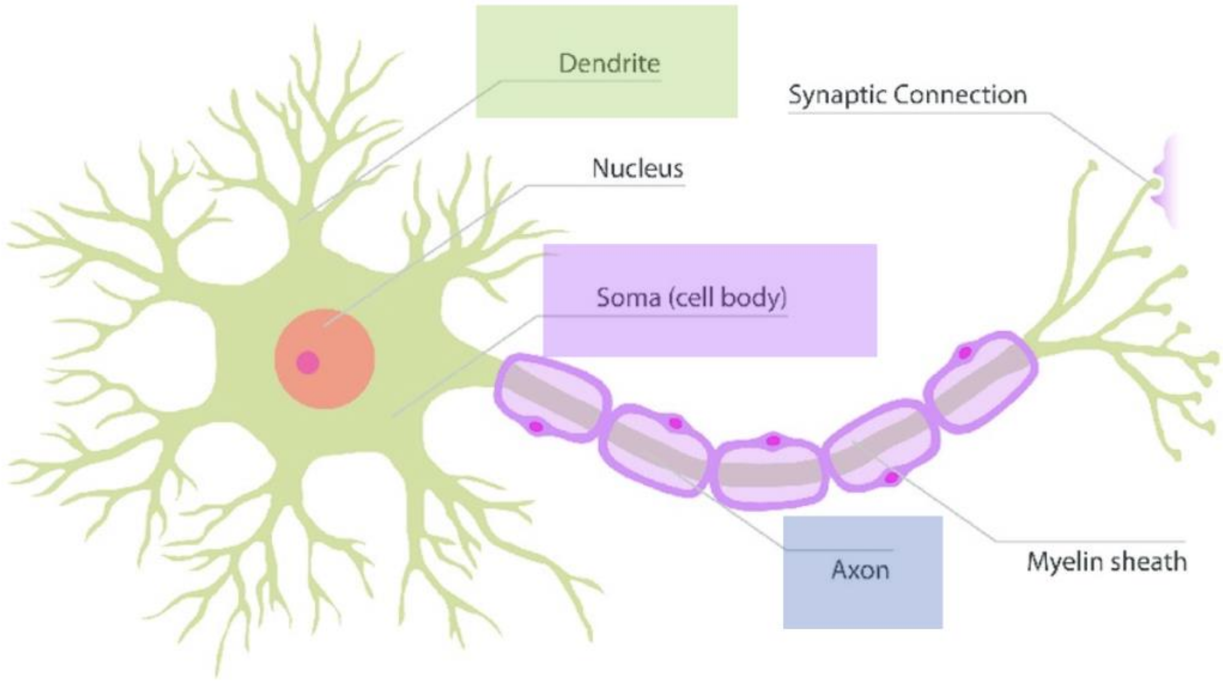
Biological Motivation

- Loosely inspired by neurons, basic working unit of the brain
- Serve to transmit information between cells

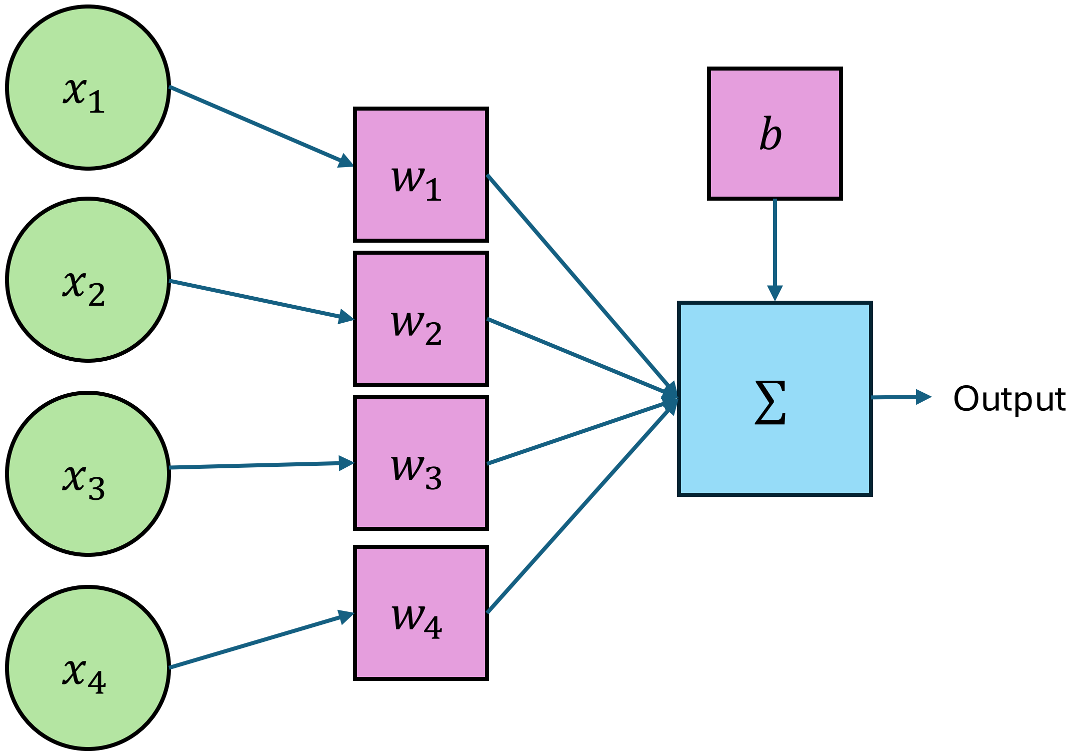




The Perceptron



Biological Neuron

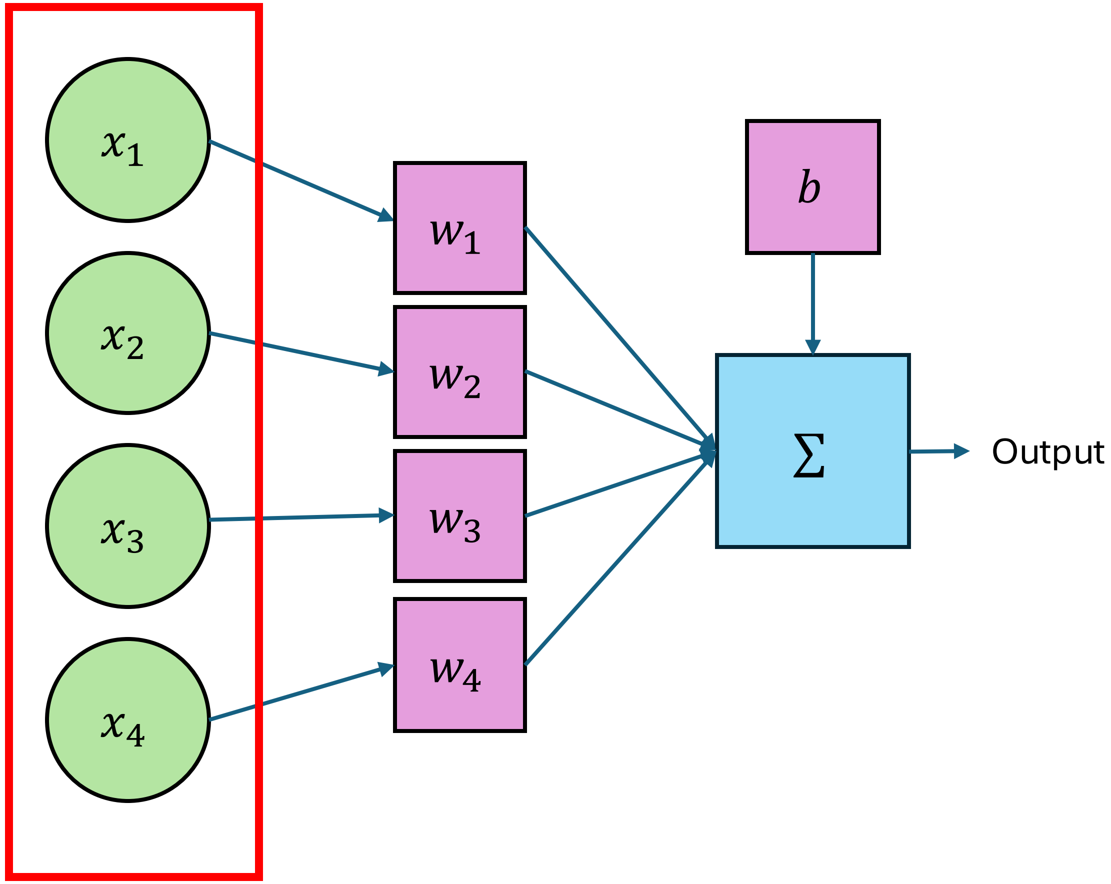


Artificial Neuron (Perceptron)

Inputs

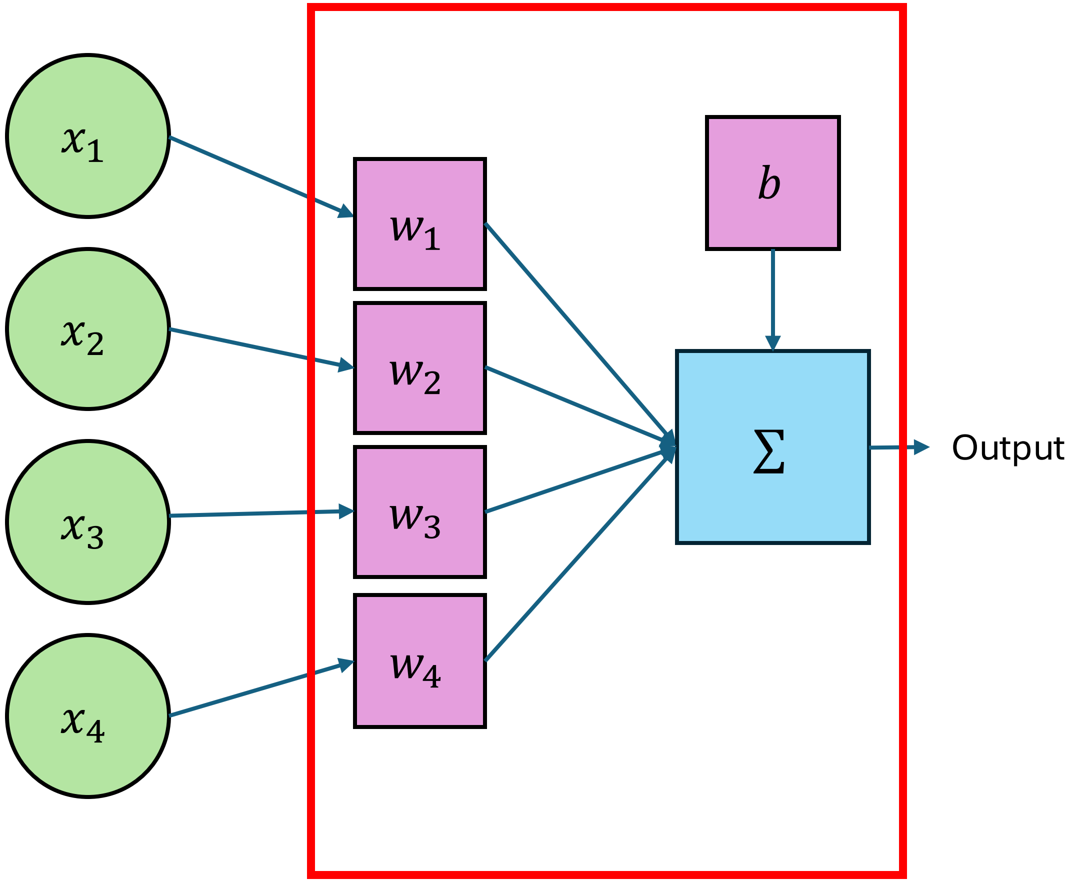
Inputs are $\vec{x} = [x_1, x_2, \ldots, x_d]$

Features of the data



Predicting with a Perceptron

1. Take each of the inputs and multiply by corresponding weight
2. Sum the results, add bias term



Predicting with a Perceptron

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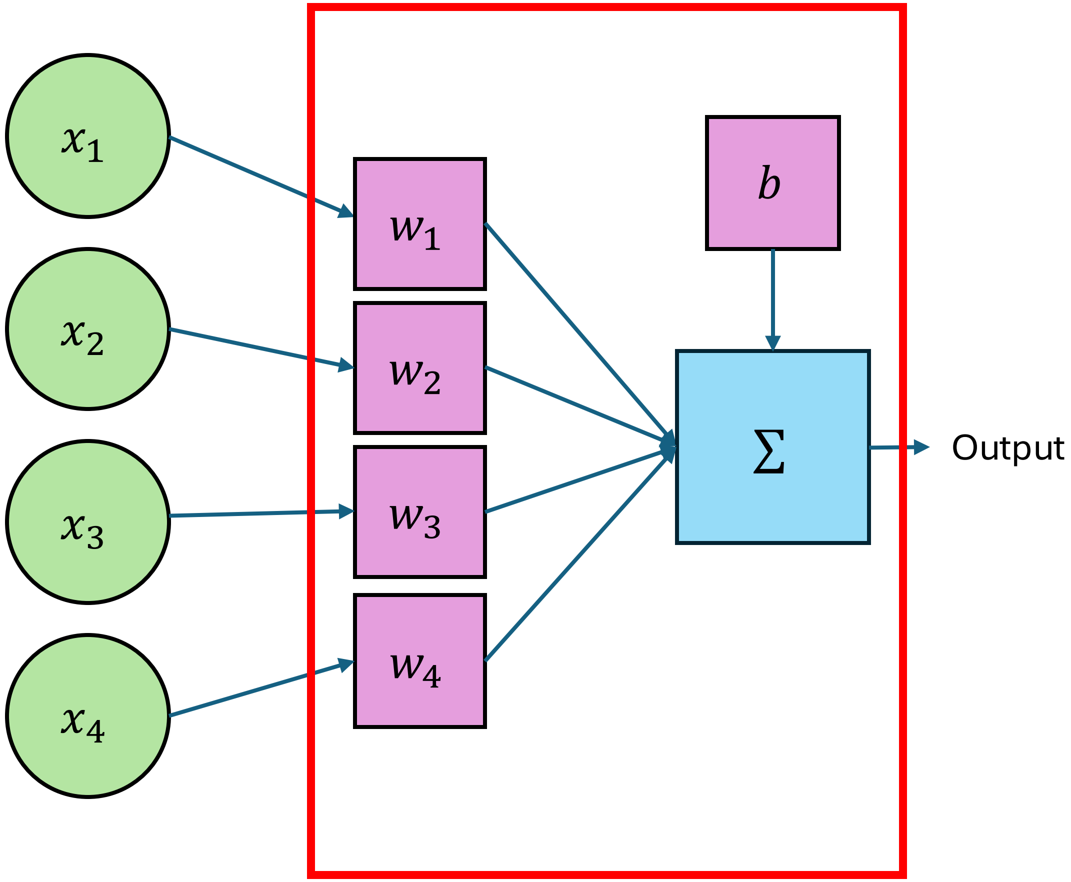
Until here, a Perceptron and Linear Regression are equivalent



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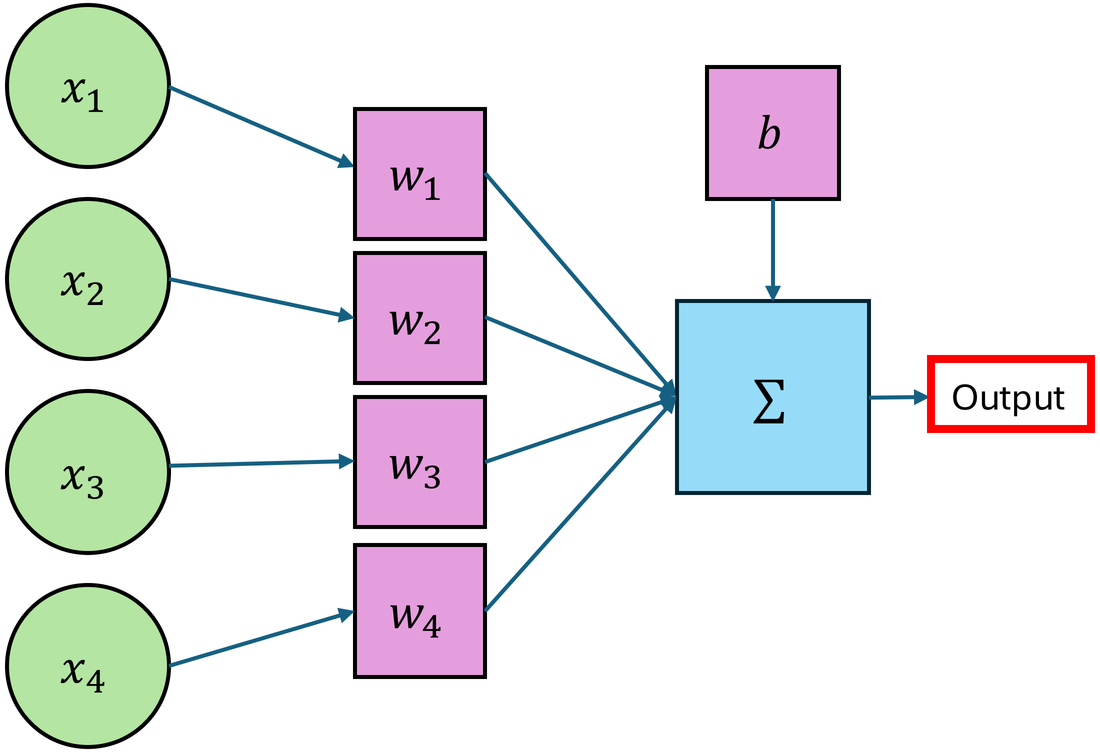

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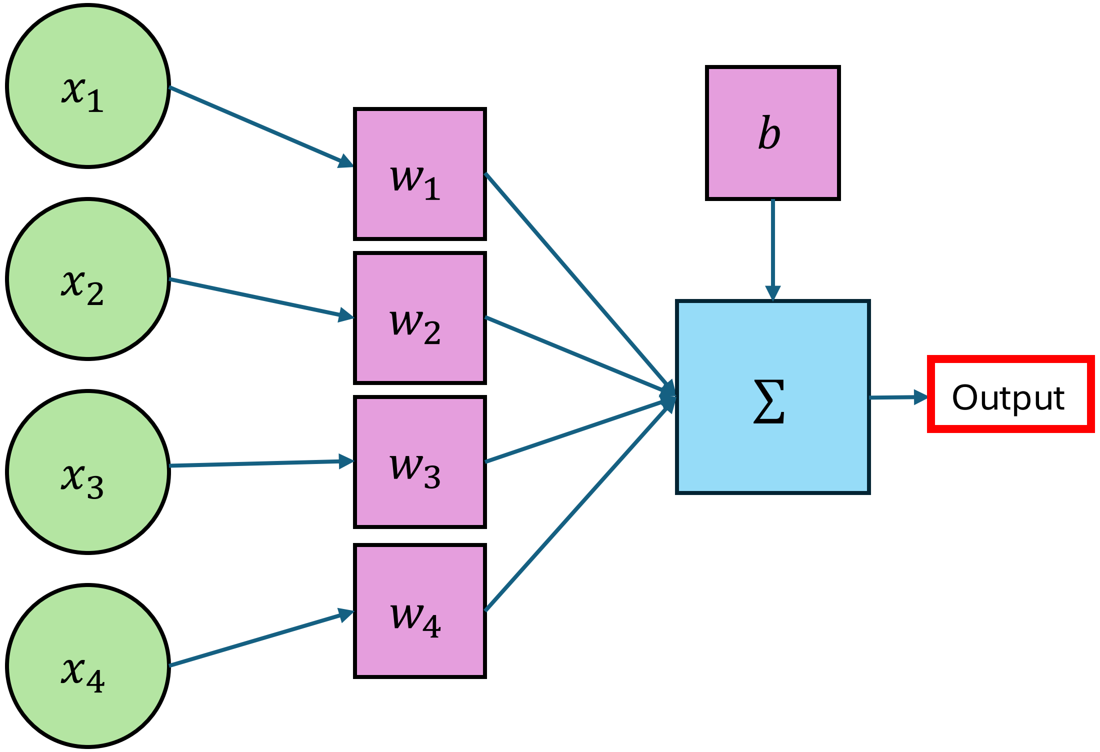
Activation Function
(many more to come)



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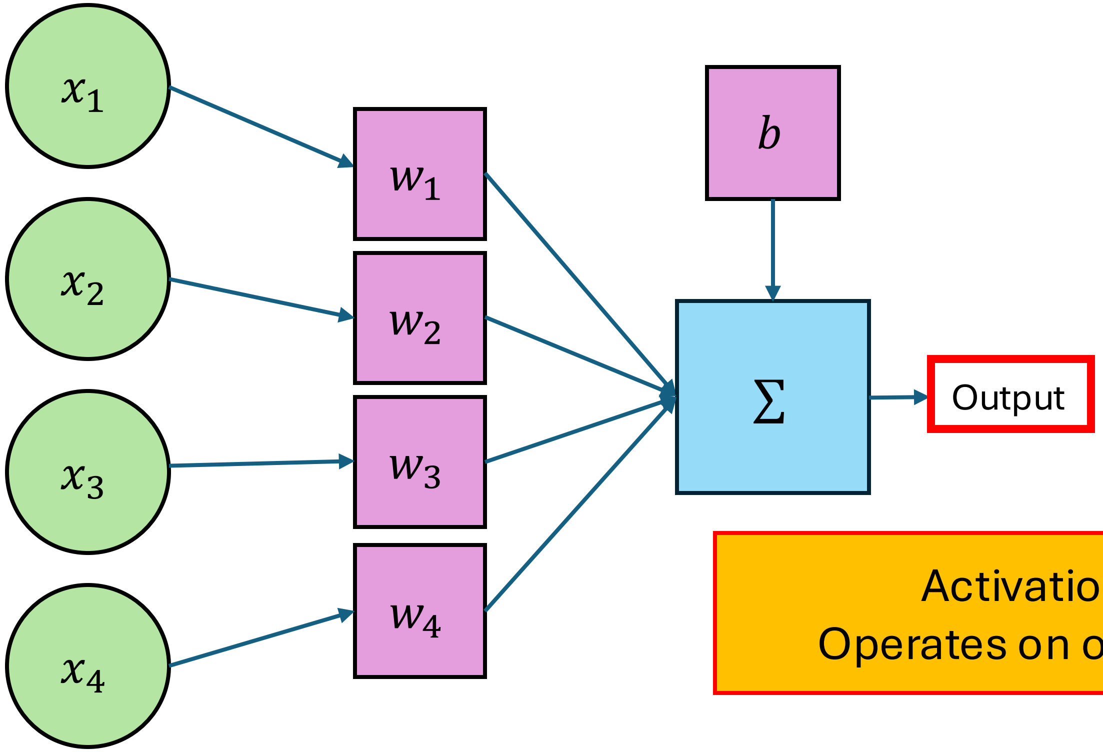
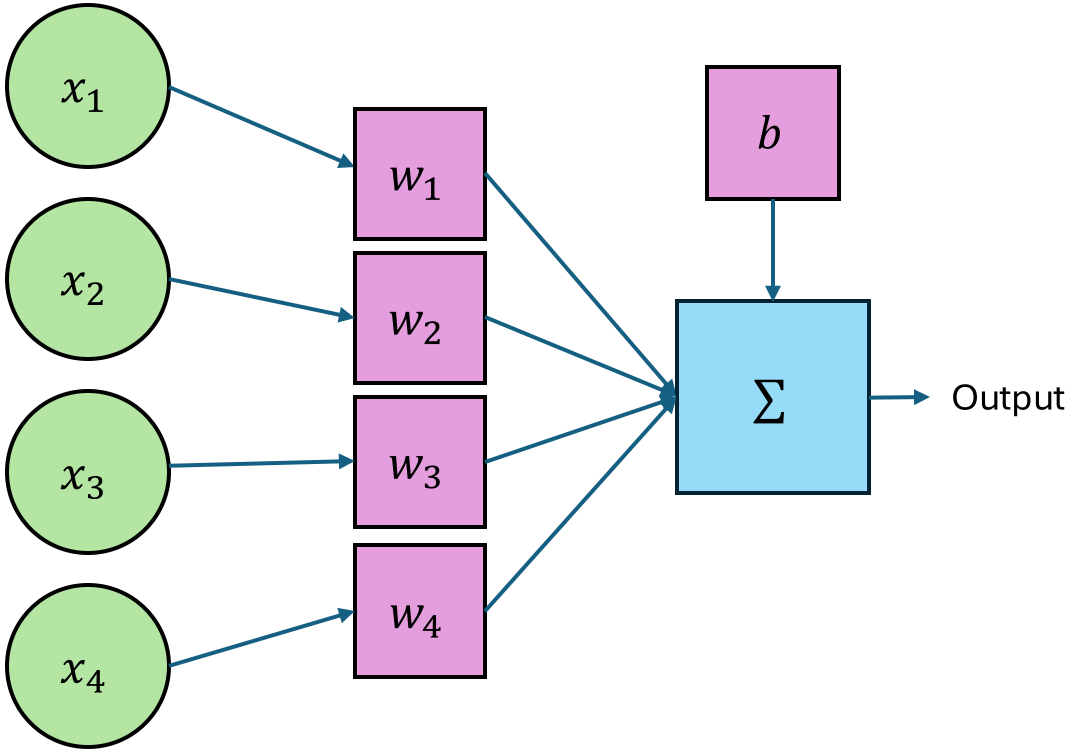


Activation Function
Operates on output of neuron

Understanding Weights



Understanding Weights

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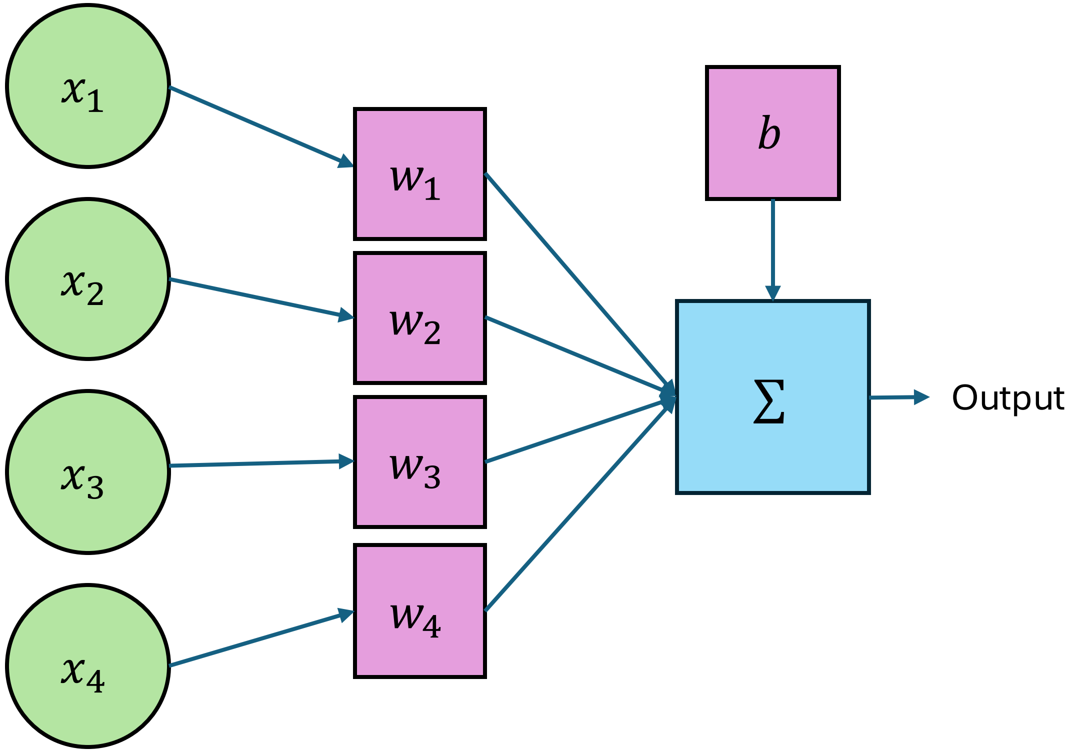

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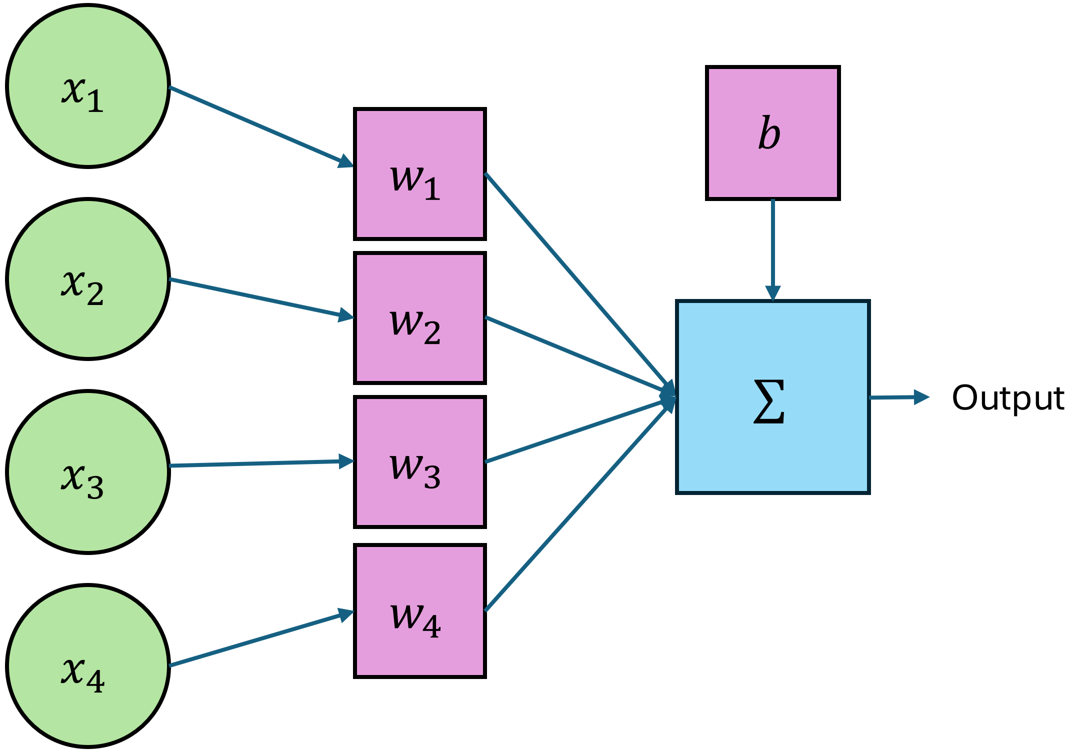

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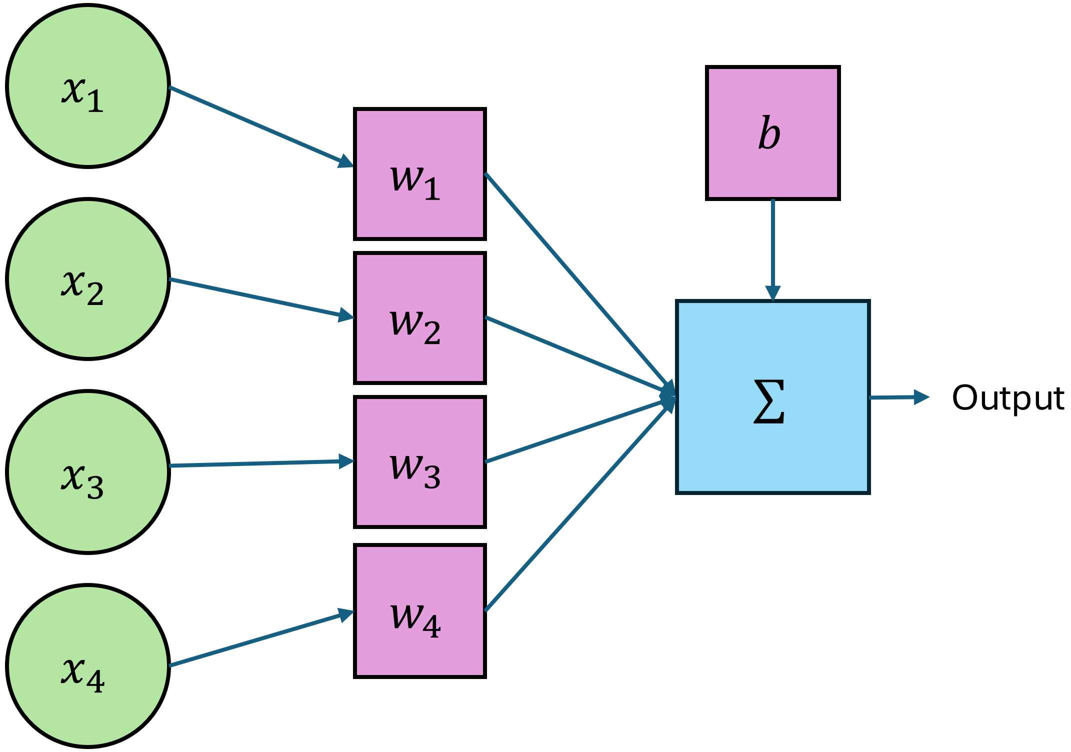

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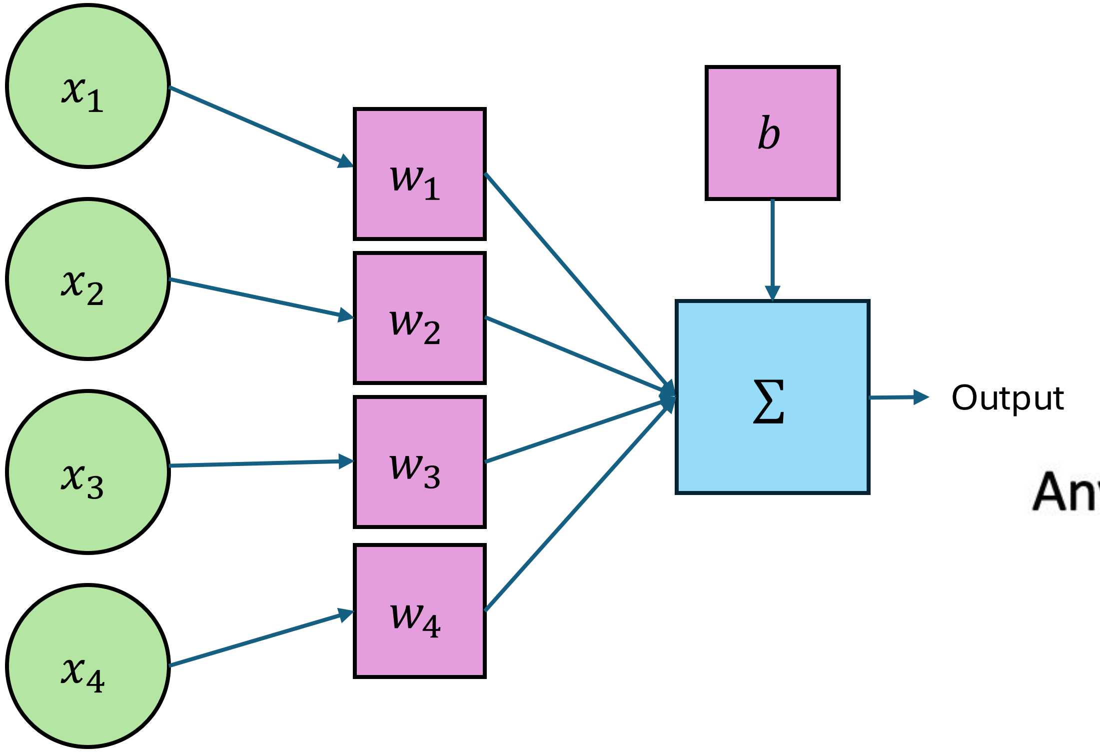
What would it mean for a weight to be very positive?

What would it mean for a weight to be very negative?



How Strong are Linear Separators?

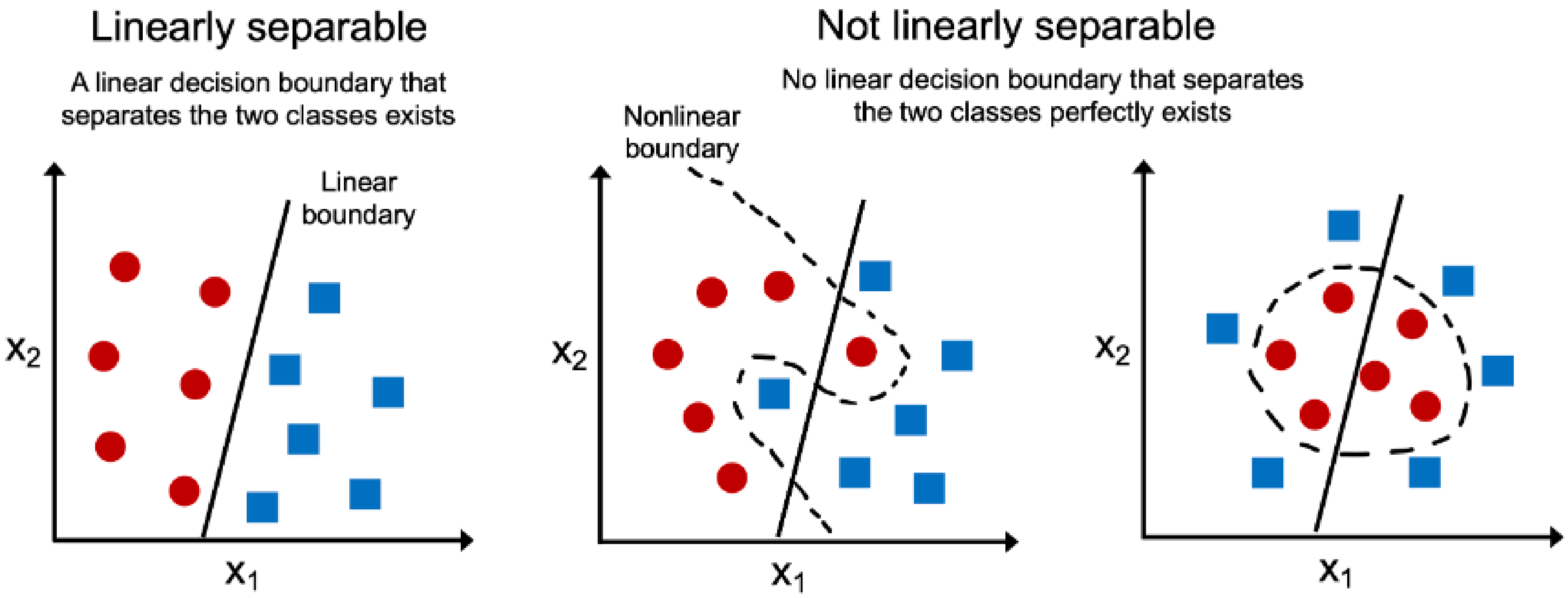


Image courtesy of: https://vitalflux.com/how-know-data-linear-non-linear/

MNIST

The most famous dataset in Deep Learning

Modified **N**ational **I**nstitute of **S**tandards and **T**echnology database

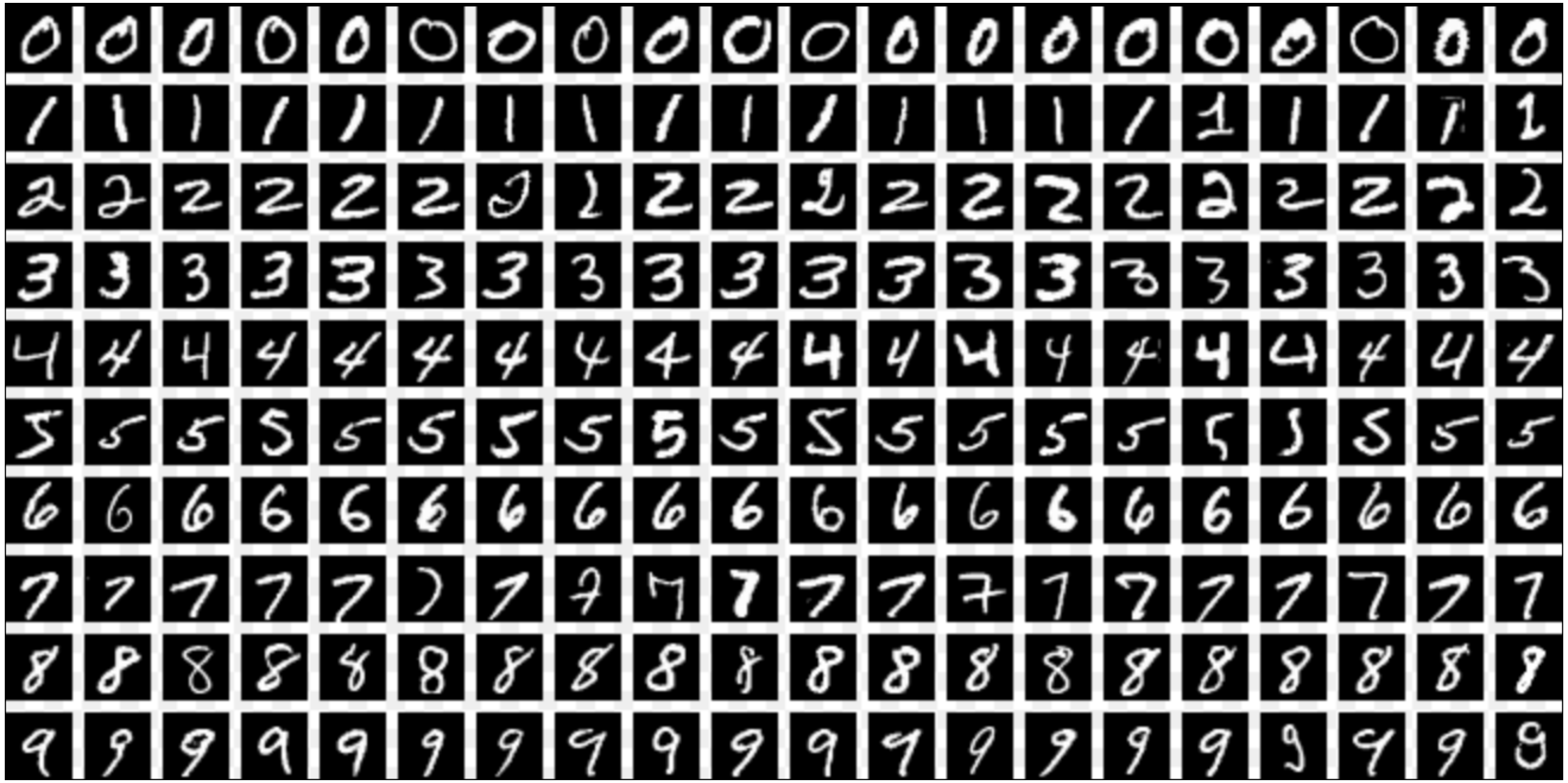
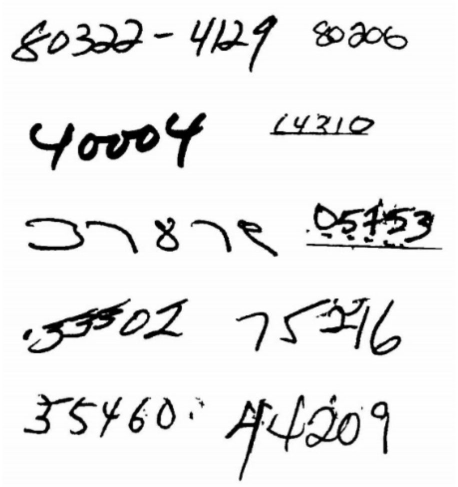


Image courtesy of Wikipedia

Motivation: Zip Code Recognition

- In 1990s, great increase in documents on paper (mail, checks, books, etc.)
- Motivation for a ZIP code recognizer on real U.S. mail for the postal service!



http://yann.lecun.com/exdb/publis/pdf/matan-92.pdf

Our Problem:

Input: $\mathbb{X}$

Function: f

$f(\mathbb{X}) \rightarrow \mathbb{Y}$

Target: $\mathbb{Y}$

Which digit is it?

“3”

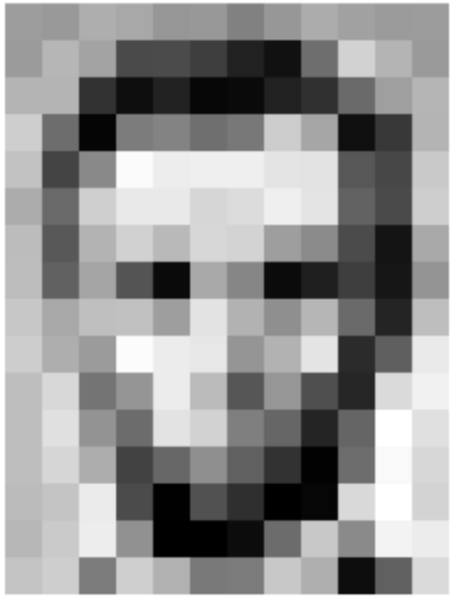
How Does a Computer know this is a three?

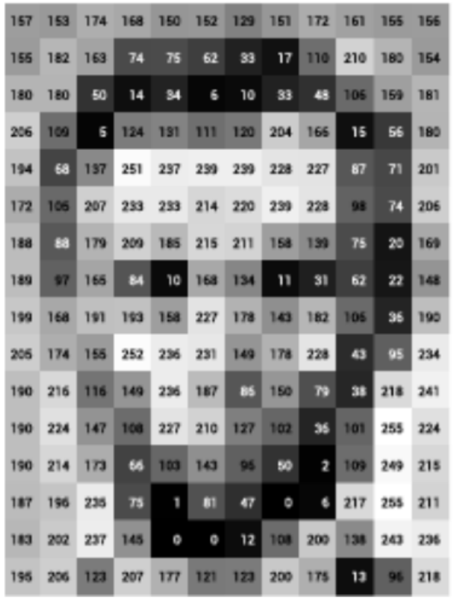
→ “three”

Representing digits in the computer

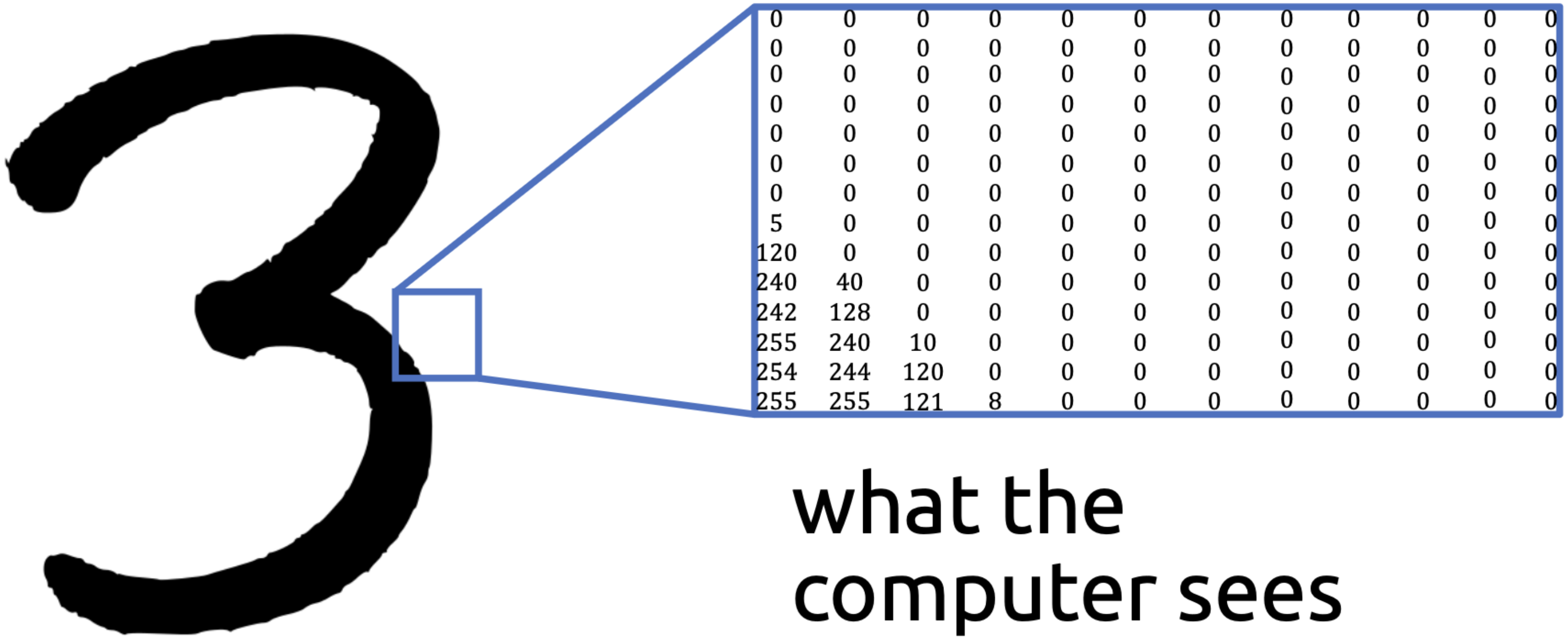
- Numbers known as ***pixel values*** (a grid of discrete values that make up an image)

0 is white, 255 is black, and numbers in between are shades of gray

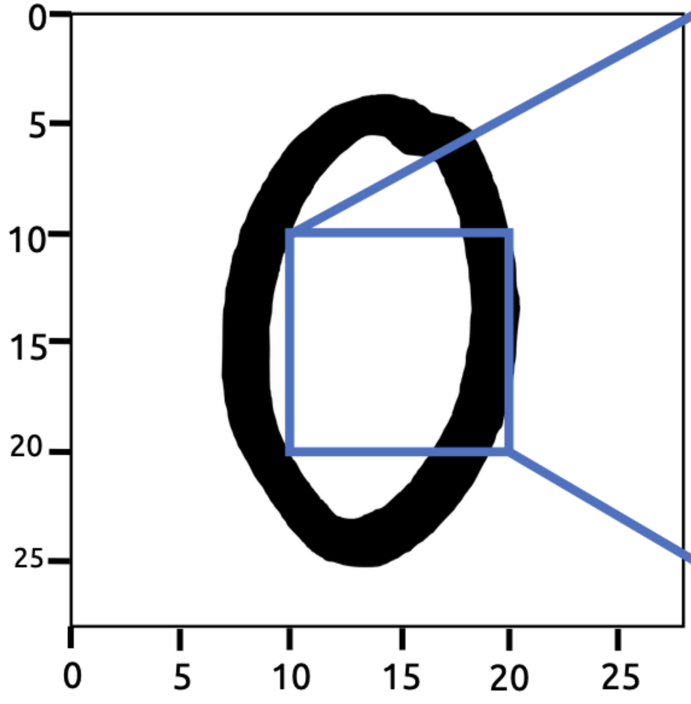




157	153	174	168	150	152	129	151	172	161	155	156
155	182	163	74	75	62	33	17	110	210	180	154
180	180	50	14	34	6	10	33	48	106	159	181
206	109	5	124	131	111	120	204	166	15	56	180
194	68	137	251	237	239	239	228	227	87	71	201
172	105	207	233	233	214	220	239	228	98	74	206
188	88	179	209	185	215	211	158	139	75	20	169
189	97	165	84	10	168	134	11	31	62	22	148
199	168	191	193	158	227	178	143	182	106	36	190
205	174	155	252	236	231	149	178	228	43	95	234
190	216	116	149	236	187	86	150	79	38	218	241
190	224	147	108	227	210	127	102	36	101	255	224
190	214	173	66	103	143	96	50	2	109	249	215
187	196	235	75	1	81	47	0	6	217	255	211
183	202	237	145	0	0	12	108	200	138	243	236
195	206	123	207	177	121	123	200	175	13	96	218



what the
computer sees

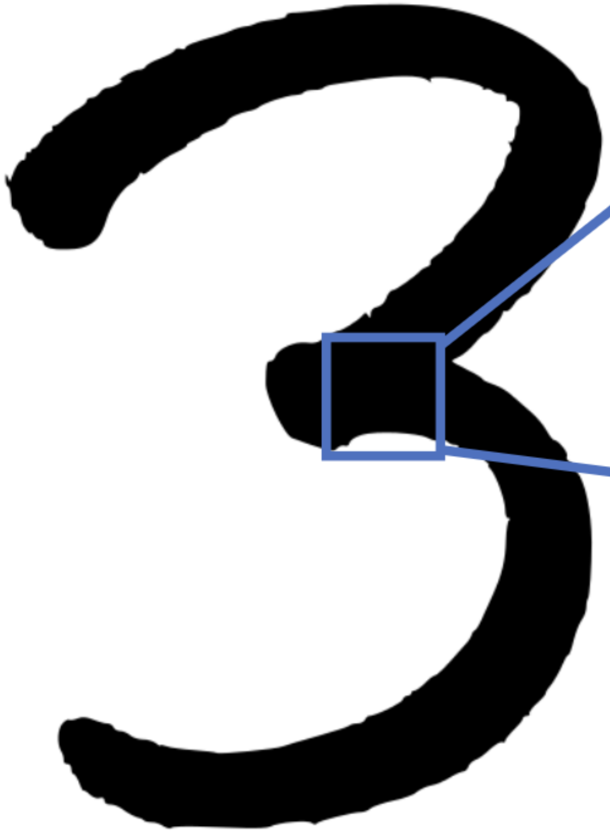


	10	11	12	13	14	15	16	17	18	19
10	0	0	0	0	0	0	0	170	236	255
11	0	0	0	0	0	0	0	0	250	254
12	0	0	0	0	0	0	0	0	255	255
13	0	0	0	0	0	0	0	0	240	245
14	0	0	0	0	0	0	0	0	245	236
15	0	0	0	0	0	0	0	0	251	253
16	0	0	0	0	0	0	0	124	255	255
17	0	0	0	0	0	0	0	210	254	120
18	0	0	0	0	0	0	0	165	230	0
19	0	0	0	0	0	0	120	154	220	0

what the computer sees

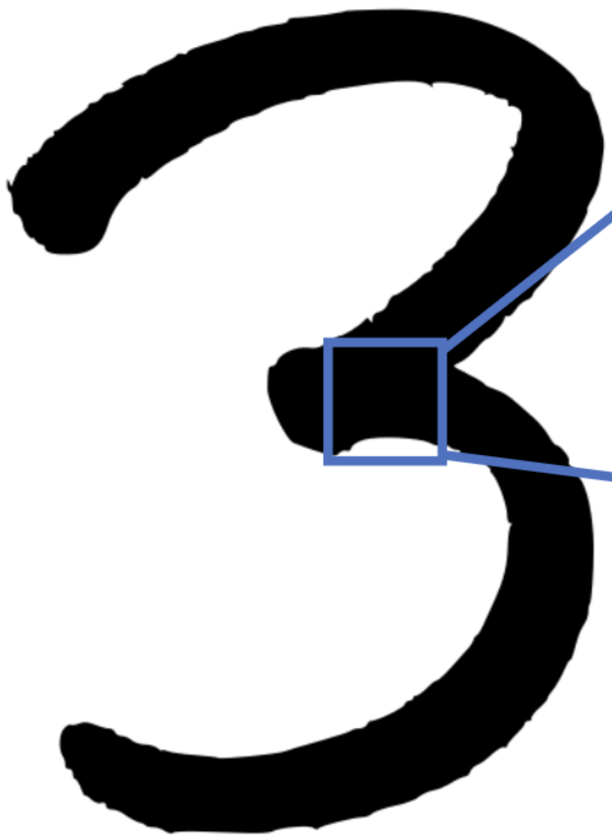
- Pixel in position [15, 15] is light.

Center is typically empty for 0’s.
How does this compare with 3’s?



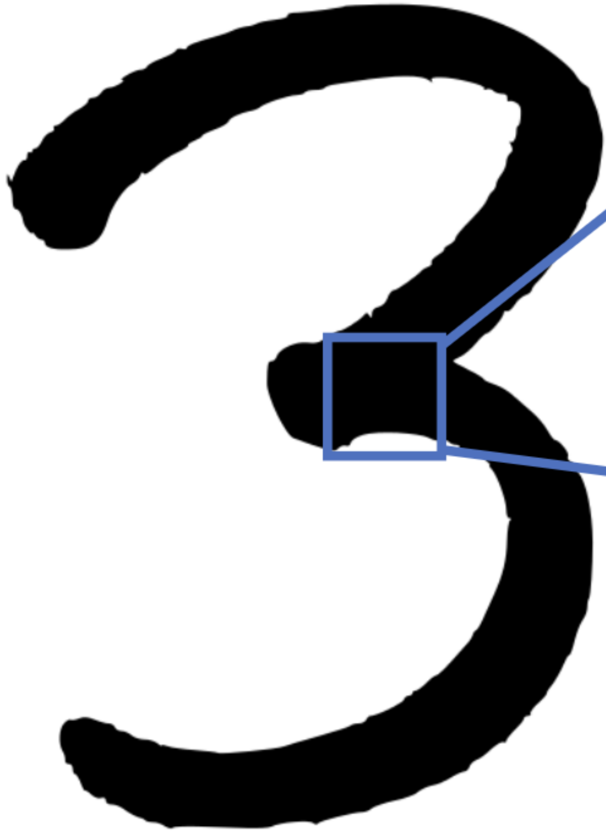
255	255	255	255	255	253	254	245	255
255	255	251	255	255	255	254	235	252
255	252	255	250	255	245	255	253	234
253	255	255	255	251	254	255	255	235
255	255	252	255	249	255	239	243	255
255	250	255	245	255	255	254	244	254
255	255	255	255	249	255	255	255	244
249	255	253	255	233	255	249	245	239
255	255	255	250	255	254	251	243	251
245	240	244	240	239	244	255	244	248
242	128	140	150	130	128	110	245	246
240	240	4	5	4	3	2	118	120
240	5	4	2	0	0	0	4	2
0	0	0	0	0	0	0	0	0

Darker pixels in the middle



255	255	255	255	255	253	254	245	255
255	255	251	255	255	255	254	235	252
255	252	255	250	255	245	255	253	234
253	255	255	255	251	254	255	255	235
255	255	252	255	249	255	239	243	255
255	250	255	245	255	255	254	244	254
255	255	255	255	249	255	255	255	244
249	255	253	255	233	255	249	245	239
255	255	255	250	255	254	251	243	251
245	240	244	240	239	244	255	244	248
242	128	140	150	130	128	110	245	246
240	240	4	5	4	3	2	118	120
240	5	4	2	0	0	0	4	2
0	0	0	0	0	0	0	0	0

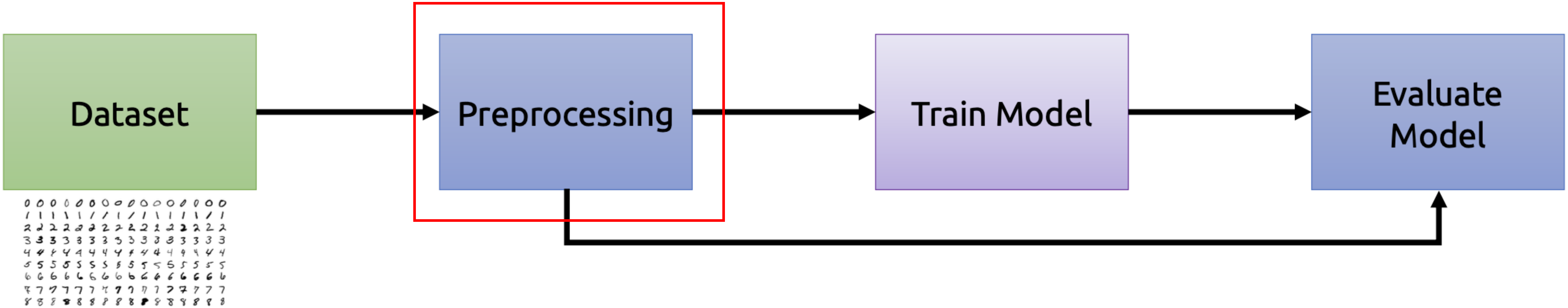
Darker pixels in the middle



255	255	255	255	255	253	254	245	255
255	255	251	255	255	255	254	235	252
255	252	255	250	255	245	255	253	234
253	255	255	255	251	254	255	255	235
255	255	252	255	249	255	239	243	255
255	250	255	245	255	255	254	244	254
255	255	255	255	249	255	255	255	244
249	255	253	255	233	255	249	245	239
255	255	255	250	255	254	251	243	251
245	240	244	240	239	244	255	244	248
242	128	140	150	130	128	110	245	246
240	240	4	5	4	3	2	118	120
240	5	4	2	0	0	0	4	2
0	0	0	0	0	0	0	0	0

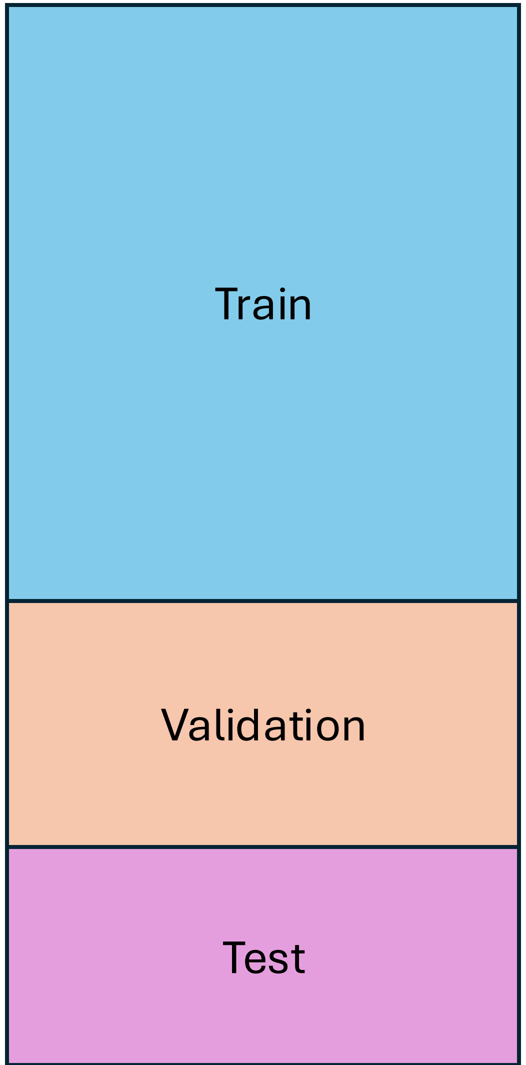
Can we define a set of ***heuristics*** (i.e. rules based on our intuition), to classify digits?

Machine Learning Pipeline for Digit Recognition



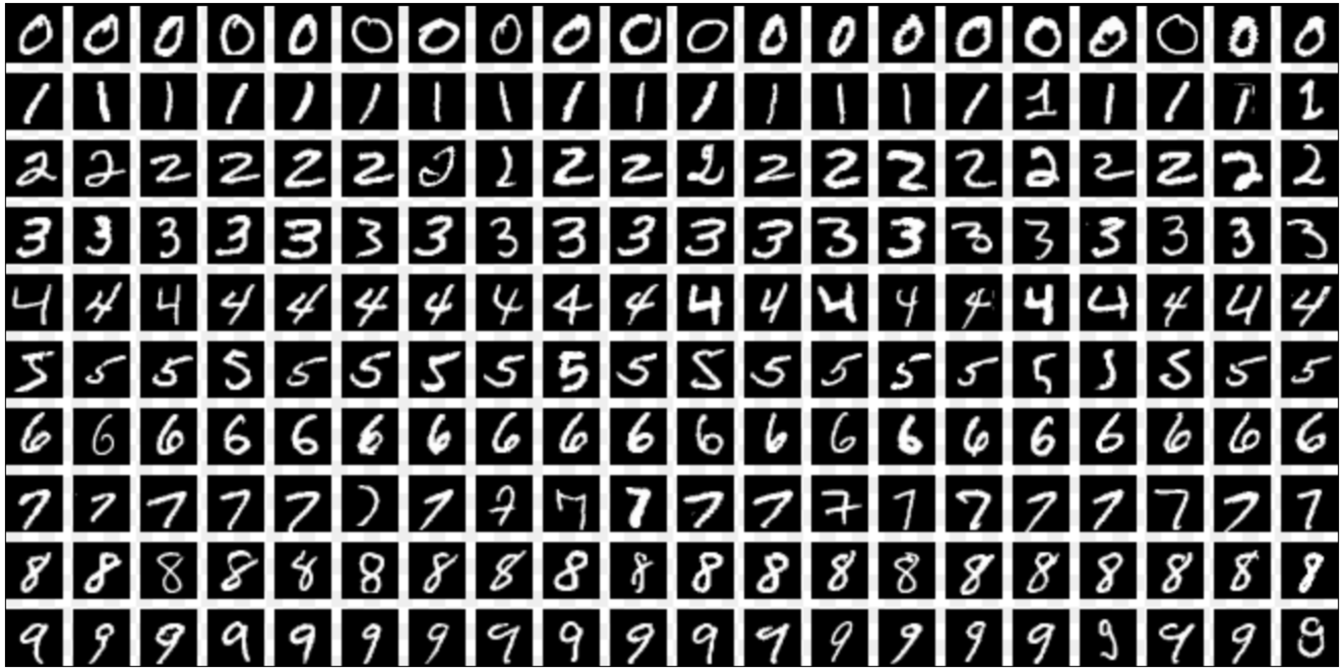
Train, validation, and test sets

- **Training Set**: Used to adjust parameters of model
- ***Validation set*** — used to test how well we're doing as we develop
  - Prevents ***overfitting***
- ***Test Set*** — used to evaluate the model once the model is done



MNIST

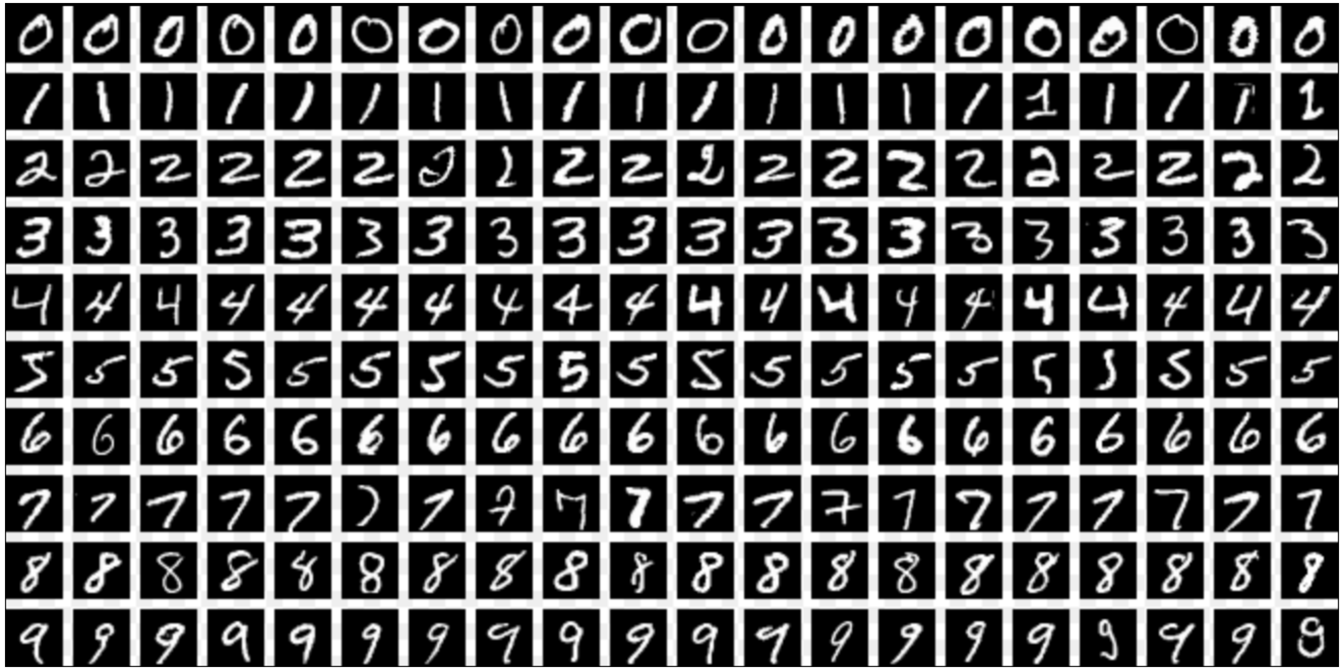
- 60,000 Images in training set
- 10,000 Images in test set
- No explicit validation set



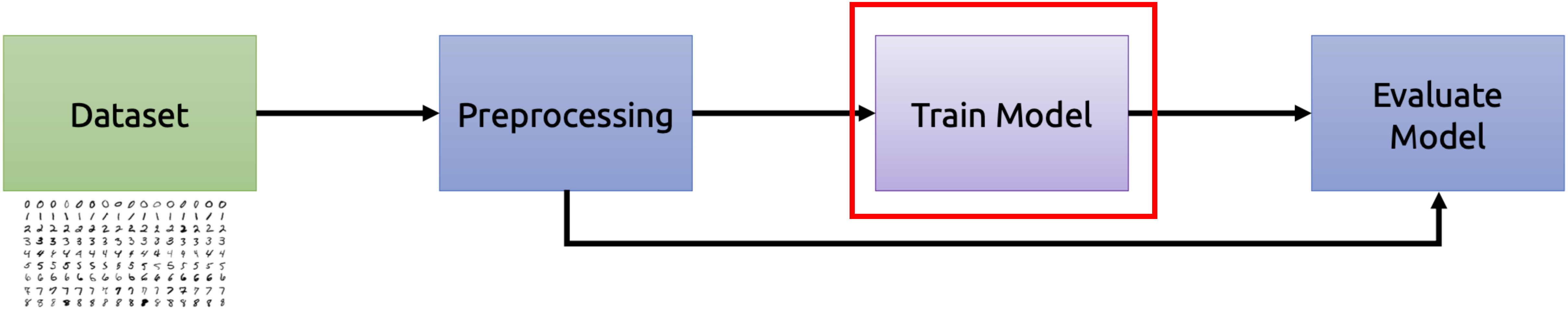
MNIST

- 60,000 Images in training set
- 10,000 Images in test set
- No explicit validation set

What do you suggest we do?



Machine Learning Pipeline for Digit Recognition



Our Problem:

Classifying MNIST digits requires predicting 1 of 10 possible values

Input: $\mathbb{X}$

Target: $\mathbb{Y}$

Pixel Grid

Which digit is it?

$x^{(1)} =$ 

28x28 pixels

Function: f

$f(\mathbb{X}) \rightarrow \mathbb{Y}$

$y^{(1)} =$ "2"

$x^{(2)} =$ 

$y^{(2)} =$ "0"

Our Problem:

Classifying MNIST digits requires predicting 1 of 10 possible values

What is our input space?

Input: $\mathbb{X}$

Target: $\mathbb{Y}$

Pixel Grid

$x^{(1)} =$

28x28 pixels

$x^{(2)} =$

→ Function: f →

$f(\mathbb{X}) \rightarrow \mathbb{Y}$

Which digit is it?

$y^{(1)} = \text{“2”}$

$y^{(2)} = \text{“0”}$

Our Problem:

Classifying MNIST digits requires predicting 1 of 10 possible values

Input: $\mathbb{X}$

What is our input space?

What is our output space?

Target: $\mathbb{Y}$

Pixel Grid

Which digit is it?

$x^{(1)} =$

28x28 pixels

Function: f

$f(\mathbb{X}) \rightarrow \mathbb{Y}$

$y^{(1)} = \text{“2”}$

$x^{(2)} =$

$y^{(2)} = \text{“0”}$

Our Problem:

Classifying MNIST digits requires predicting 1 of 10 possible values

Input: $\mathbb{X}$

What is our input space?

What is our output space?

What is our prediction task?

Target: $\mathbb{Y}$

Pixel Grid

Which digit is it?

$x^{(1)} =$ 

Function: f

$y^{(1)} =$ "2"

28x28 pixels

$f(\mathbb{X}) \rightarrow \mathbb{Y}$

$x^{(2)} =$ 

$y^{(2)} =$ "0"

Our simplified problem:

Input: $\mathbb{X}$

What is our input space?

What is our output space?

What is our prediction task?

Target: $\mathbb{Y}$

Pixel Grid

Is it digit 2?

$x^{(1)} =$ 

28x28 pixels

Function: f

$f(\mathbb{X}) \rightarrow \mathbb{Y}$

$y^{(1)} = 1$ 

$x^{(2)} =$ 

$y^{(2)} = 0$ 

A bit of a cliffhanger…

- How well do you think a perceptron will do on this task?
- Perceptrons are linear classifiers… what does it mean for images to be linearly separable?
- Perceptrons have a discontinuous activation function, which is not differentiable. How are we going to find good parameters without a nice closed-form solution?

Recap

Linear Regression

- Matrix and Vector Notation
- Closed Form solution for finding optimal parameters

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Perceptrons

- natural extension of linear models to binary classification tasks
- Biological inspiration of activation threshold
- Only differ from Linear Regression in terms of activation function

Recap

Linear Regression

- Matrix and Vector Notation
- Closed Form solution for finding optimal parameters

Perceptrons

- natural extension of linear models to binary classification tasks
- Biological inspiration of activation threshold
- Only differ from Linear Regression in terms of activation function

MNIST

- Handwritten Digit Representation
- Handwriting Classification Task Framing